

## Quiz 1

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1. By a *branch of the argument* we mean

- A. specifying a particular range for the angle.
  - B. allowing the addition of  $2\pi$  to the angle.
  - C. allowing any value for the angle.
  - D. All of the above.
  - E. None of the above.
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2. If  $z_1, z_2$  are nonzero complex numbers then

- A.  $\arg(z_1 + z_2) = \arg(z_1) + \arg(z_2)$ .
  - B.  $\arg(z_1 z_2) = \arg(z_1) \arg(z_2)$ .
  - C.  $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$ .
  - D. All of the above.
  - E. None of the above.
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3. If  $z_1, z_2$  are complex numbers then

- A.  $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ .
  - B.  $\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$ .
  - C.  $z_1 \overline{z_1} = |z_1|^2$ .
  - D. All of the above.
  - E. None of the above.
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4. If  $z_1, z_2$  are complex numbers then

- A.  $|z_1 + z_2| \leq |z_1| + |z_2|$ .
  - B.  $|z_1 - z_2| \geq ||z_1| - |z_2||$ .
  - C.  $-|z_1| \leq \operatorname{Re}(z_1) \leq |z_1|$ .
  - D. All of the above.
  - E. None of the above.
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5. De Moivre's Theorem says that if  $z$  is a complex number and  $n$  is a positive integer then

- A.  $|z^n| = |z|^n$  and  $\arg(z^n) = n \arg(z) \bmod 2\pi$ .
- B.  $|z^n| = n |z|$  and  $\arg(z^n) = \arg(z)^n \bmod 2\pi$ .
- C.  $|z^n| = |z|$  and  $\arg(z^n) = \arg(z) \bmod 2\pi$ .
- D. All of the above.
- E. None of the above.

## Quiz 2

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1. A function  $f: A \rightarrow \mathbb{C}$  is *uniformly continuous* on  $A$  if

- A. for every  $\epsilon > 0$  there is a  $\delta > 0$  such that  $|f(s) - f(t)| < \epsilon$  whenever  $s, t \in A$  and  $|s - t| < \delta$ .
  - B. for every  $s \in A$  and  $\epsilon > 0$  there is a  $\delta > 0$  such that  $|f(s) - f(t)| < \epsilon$  whenever  $t \in A$  and  $|s - t| < \delta$ .
  - C. for every  $\epsilon > 0$  there is a  $\delta > 0$  such that  $|f(s) - f(t)| < \epsilon$  implies  $s, t \in A$  and  $|s - t| < \delta$ .
  - D. All of the above.
  - E. None of the above.
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2. If  $K$  is compact then

- A. every open cover of  $K$  has a finite subcover.
  - B. every sequence in  $K$  has a subsequence that converges to a point in  $K$ .
  - C.  $K$  is closed and bounded.
  - D. All of the above.
  - E. None of the above.
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3. If  $f$  is continuous on the closed unit disk  $U$  then

- A.  $f$  is uniformly continuous on  $U$ .
  - B.  $|f(z)|$  attains a maximum value somewhere on  $U$ .
  - C. the image of  $U$  is connected.
  - D. All of the above.
  - E. None of the above.
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4. If  $\gamma: [a, b] \rightarrow \mathbb{C}$  is a smooth path then  $-\gamma$  denotes the path defined by the formula

- A.  $-\gamma(t)$ .
  - B.  $\gamma(-t)$ .
  - C.  $-\gamma(-t)$ .
  - D. All of the above.
  - E. None of the above.
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5. If  $\gamma(t) = e^{it}$  for  $t \in [0, 2\pi]$  then  $\int_{\gamma} |dz|$  equals

- A. 1.
- B.  $2\pi$ .
- C. 0.
- D. All of the above.
- E. None of the above.

### Quiz 3

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1. Suppose  $f$  is analytic on a region  $G$ , and  $\gamma$  is a piecewise smooth, closed path in  $G$ . Under what conditions is  $\int_{\gamma} f = 0$ ?

- A.  $\gamma$  is homotopic to a point in  $G$ .
  - B.  $f$  has an antiderivative in  $G$ .
  - C.  $\gamma$  lies in a disk that is contained in  $G$ .
  - D. All of the above.
  - E. None of the above.
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2. Suppose  $f$  defined on a disk  $D_r(z_0)$  and analytic and on the deleted disk. Under what conditions is  $f$  also analytic at  $z_0$ ?

- A.  $f$  is bounded on the disk.
  - B.  $f$  is continuous on the disk.
  - C.  $\lim_{z \rightarrow z_0} f(z)(z - z_0) = 0$ .
  - D. All of the above.
  - E. None of the above.
- 

3. Suppose  $\gamma$  is a closed path in some deleted disk around a point  $z_0$ . Under what conditions is  $I(\gamma, z_0) = 0$ ?

- A.  $\gamma$  is homotopic to a point in the undeleted disk.
  - B.  $\gamma$  is homotopic to a point in the deleted disk.
  - C.  $\gamma$  is homotopic to a point in  $\mathbb{C}$ .
  - D. All of the above.
  - E. None of the above.
- 

4. Suppose  $f$  is analytic in a region  $G$ . Which of the following is true?

- A. For every piecewise smooth, closed curve in  $G$ ,  $\int_{\gamma} f = 0$ .
  - B.  $f$  has an antiderivative in  $G$ .
  - C.  $f'$  is continuous in  $G$ .
  - D. All of the above.
  - E. None of the above.
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5. Suppose  $f$  is continuous in a region  $G$  and that  $\int_{\gamma} f = 0$  for every piecewise smooth, closed path in  $G$ . Which of the following is true?

- A. Path integrals of  $f$  in  $G$  depend only on the endpoints.
- B.  $f$  has an antiderivative in  $G$ .
- C.  $f$  is analytic in  $G$ .
- D. All of the above.
- E. None of the above.

## Quiz 4

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1. If  $\sum a_n z^n$  converges at a point  $z_1$  then the series

- A. converges at every  $z$  such that  $|z| < |z_1|$ .
  - B. converges uniformly on the closed disk  $\{z \mid |z| \leq |z_1|\}$ .
  - C. diverges at every  $z$  such that  $|z| > |z_1|$ .
  - D. All of the above.
  - E. None of the above.
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2. If  $f_n(z)$  are analytic on a region  $A$  and  $f_n \rightarrow f$  uniformly on every compact  $K$  in  $A$  then

- A.  $f$  is uniformly continuous on every compact  $K$  in  $A$ .
  - B.  $f$  is analytic on  $A$ .
  - C.  $f'_n \rightarrow f'$  uniformly on every compact  $K$  in  $A$ .
- D. All of the above.  
E. None of the above.
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3. If the partial sums of  $\sum z_n$  form a Cauchy sequence then

- A. the partial sums of  $\sum |z_n|$  form a Cauchy sequence.
  - B.  $\sum |z_n|$  converges.
- C.  $\sum z_n$  converges.  
D. All of the above.  
E. None of the above.
- 

4. If  $\sum z_n$  is an absolutely convergent series then

- A. the partial sums of  $\sum |z_n|$  form a Cauchy sequence.
  - B.  $\sum |z_n|$  converges.
  - C.  $\sum z_n$  converges.
- D. All of the above.
- 

5. Which of the following, if it exists, is the radius of convergence for  $\sum a_n z^n$ ?

- A.  $\lim_{n \rightarrow \infty} |a_{n+1}| / |a_n|$ .
- B.  $\lim_{n \rightarrow \infty} |a_n| / |a_{n+1}|$ .
- C.  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$ .
  - D. All of the above.
  - E. None of the above.

## Quiz 5

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1. Suppose  $f$  is analytic on the disk  $D_r(z_0)$ . Consider the power series for  $f$  at  $z_0$ . Which of the following is true?
- A. The radius of convergence is exactly  $r$ .
  - B. The radius of convergence is at most  $r$ .
  - C. The radius of convergence is at least  $r$ .
  - D. None of the above.
- 
2. Suppose  $z_0$  is an isolated singularity for  $f$ , and  $\lim_{z \rightarrow z_0} (z - z_0)f(z) = 0$ . Which of the following is true?
- A.  $z_0$  is a removable singularity.
  - B.  $z_0$  is a simple pole.
  - C.  $z_0$  is a pole, but not necessarily simple.
  - D. None of the above.
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3. Suppose  $z_0$  is an isolated singularity for  $f$ , and  $\lim_{z \rightarrow z_0} (z - z_0)f(z) = L \neq 0$ . Which of the following is true?
- A.  $z_0$  is a removable singularity.
  - B.  $z_0$  is a simple pole, and  $L$  is the residue.
  - C.  $z_0$  is a pole, but not necessarily simple.
  - D. None of the above.
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4. Suppose  $z_0$  is an isolated singularity for  $f$ , and  $\lim_{z \rightarrow z_0} f(z) = \infty$ . Which of the following is true?
- A.  $z_0$  is an essential singularity.
  - B.  $z_0$  is a simple pole.
  - C.  $z_0$  is a pole, but not necessarily simple.
  - D. None of the above.
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5. What are we saying about the singularities of  $f$  when we say that  $f$  is meromorphic in a region  $A$ ?
- A. All are isolated and removable.
  - B. All are isolated and at worst simple poles.
  - C. All are isolated and none are essential.
  - D. None of the above.

## Quiz 6

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1. If  $f$  is a conformal map then

- A.  $f$  preserves (oriented) angles.
  - B.  $f$  maps circles and lines to circles and lines.
  - C.  $f$  preserves cross ratios.
  - D. All of the above.
  - E. None of the above.
- 

2. If  $f$  is a linear fractional transformation then

- A.  $f$  preserves (oriented) angles.
  - B.  $f$  maps circles and lines to circles and lines.
  - C.  $f$  preserves cross ratios.
  - D. All of the above.
  - E. None of the above.
- 

3. True or false: If  $C_1$  and  $C_2$  are circles then there is a linear fractional transformation that maps  $C_1$  onto  $C_2$ .

- A. True.
  - B. False.
- 

4. True or False: If  $G$  is a simply connected region in the plane and  $G \neq \mathbb{C}$  then there is a conformal equivalence of  $G$  onto the unit disk.

- A. True.
  - B. False.
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5. True or False: If  $T$  is a bijective analytic map from the unit disk to itself, then  $T$  is a linear fractional transformation.

- A. True.
- B. False.

## Quiz 7

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1. True or False: If  $f$  can be analytically continued in a region  $A$  along any two paths from  $z_0$  to  $z_1$  then the radius of convergence of the power series at  $z_1$  is always the same, no matter what continuation is used.

- A. True.  
B. False.
- 

2. True or False: If  $f$  can be analytically continued in a region  $A$  along any two paths from  $z_0$  to  $z_1$  then the value of the function so defined at  $z_1$  is always the same, no matter what continuation is used.

- A. True.  
→B. False.
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3. True or False: If  $f$  and  $g$  are analytic on  $A$  and  $B$ , respectively, and if  $f$  and  $g$  agree on  $A \cap B$ , then there is an analytic function on  $A \cup B$  which agrees with  $f$  on  $A$  and  $g$  on  $B$ .

- A. True.  
B. False.
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4. True or False: If  $A$  is simply connected, if  $f$  is analytic on a neighborhood of  $z_0$  in  $A$ , and if  $f$  can be analytically continued to any other point of  $A$ , then analytic continuation defines an unambiguous function which is analytic on all of  $A$ .

- A. True.  
B. False.
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5. True or False: If a power series based at  $z_0$  has a finite radius of convergence  $R$  then there is a point  $z_1$  at distance  $R$  from  $z_0$  to which the function defined by the power series cannot be analytically continued.

- A. True.  
B. False.