Quiz 1

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1. What is an integral curve?

A. A differential equation whose solution is a curve.
B. An integral whose graph is a differential equation.
→*C*. The graph of a solution to a differential equation.
D. All of the above.
E. None of the above.

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2. Which of the following is an example of a homogeneous third-order linear differential equation?

A. $y''' + ty'' + 3y = \sin(t)$.
B. $yy''' + 2y = 0$.
C. $y'' + 3y' - 2y = 0$.
D. All of the above.
→*E*. None of the above.

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3. Which of the following is a solution to the differential equation $y' = 3t^2y$?

A. $y = 3e^{t^3}$.
B. $y = -3e^{t^3}$.
C. $y = e^{t^3}$.
→*D*. All of the above.
E. None of the above.

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4. What is an isocline for the differential equation $y' = f(t, y)$?

→*A*. The set of points where $y' = k$, for some constant k.
B. The set of critical curves for the differential equation.
C. The set of points where the solution is valid.
D. All of the above.
E. None of the above.

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5. What equation describes the critical curve for the differential equation $y' = t - y^2$?

A. $y^2 = 0$.
→*B*. $t = y^2$.
C. $t = 0$.
D. All of the above.
E. None of the above.

Quiz 2

1. True or false: Every first-order linear equation is autonomous.
False.

2. True or false: Every first-order autonomous equation is linear.
False.

3. True or false: Any two solutions to a nonhomogeneous linear equation differ by a solution to the homogeneous equation.
True.

4. True or false: Every first-order autonomous equation is separable.
True.

5. True or false: Every first-order linear homogeneous equation is separable.
True.

Quiz 3

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1. What is an equilibrium value for an autonomous differential equation?

→*A*. A constant solution.
B. The interval of validity.
C. The asymptotic behavior.
D. All of the above.
E. None of the above.

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2. Which of the following has a stable equilibrium at 0?

→*A*. $y' = -y(y+1)$.
B. $y' = y(y+1)$.
C. $y' = -y(y-1)$.
D. All of the above.
E. None of the above.

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3. If $y' = f(y)$, $f < 0$, and $f' > 0$ then

A. y is increasing and concave up.
B. y is increasing and concave down.
C. y is decreasing and concave up.
D. All of the above.
→*E*. None of the above.

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4. What is the interval of validity for the solution of $y' + ty = e^{-t}\sin(2t)$?

→*A*. $(-\infty, +\infty)$
B. $(-\infty, 0)$
C. $(0, +\infty)$
D. It cannot be determined from the given information.

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5. What is the interval of validity for the solution of $y' = y^2 + 1$?

A. $(-\infty, +\infty)$
B. $(-\infty, 0)$
C. $(0, +\infty)$
→*D*. It cannot be determined from the given information.

Quiz 4

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1. What integral equation is equivalent to the initial value problem $y' = t^2 - y$, $y(-1) = 2$?

A. $y(t) = -1 + \int_2^t (s^2 - y(s))\, ds$.

→*B.* $y(t) = 2 + \int_{-1}^t (s^2 - y(s))\, ds$.

C. $y(t) = 2 + \int_{-1}^t sy(s)\, ds$.

D. All of the above.

E. None of the above.

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2. What do we get when we apply Euler's Method with stepsize $h = 0.1$ to the initial value problem $y' = f(t, y)$, $y(2) = -1$?

A. A formula for an approximation to the solution of the initial value problem, valid on the interval $(2 - h, 2 + h)$.

→*B.* Approximate values for the solution of the initial value problem at the points $2, 2+h, 2+2h, 2+3h, \ldots$.

C. Approximate values for the solution of the initial value problem at certain equally spaced points in the interval $(2 - h, 2 + h)$.

D. All of the above.

E. None of the above.

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3. What is the difference between absolute error and relative error?

A. The absolute error is the absolute value of the relative error.

B. The relative error is the absolute error as a percentage of the stepsize.

→*C.* The relative error is the ratio of the absolute error to the correct value.

D. All of the above.

E. None of the above.

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4. True or false: the relative error in Euler's Method decreases at roughly the same rate as we decrease the stepsize.

→*A.* True.

B. False.

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5. True or false: If $f(t, y)$ and $\partial f/\partial y$ are continuous in an interval containing the point t_0 then there is unique solution to the initial value problem $y' = f(t, y)$, $y(t_0) = y_0$, which is valid on that same interval.

A. True.

→*B.* False.

Quiz 5

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1. Suppose $\vec{x}$ is an eigenvector for a 2×2 constant matrix A, and r is the corresponding eigenvalue. Which of the following is true?

A. $A\vec{x} = r\vec{x}$.
B. $r^2 - \text{tr}(A)r + \det(A) = 0$.
C. $e^{rt}\vec{x}$ is a solution to the system $\vec{w}' = A\vec{w}$.
$\rightarrow D$. All of the above.
E. None of the above.

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2. Suppose r_1, r_2 are the eigenvalues for a 2×2 constant matrix A. If $r_1 < 0$ and $r_2 > 0$ then the equilibrium for the system $\vec{w}' = A\vec{w}$ is

A. an unstable node.
B. a stable node.
$\rightarrow C$. a saddle point.
D. none of the above.

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3. Suppose r_1, r_2 are the eigenvalues for a 2×2 constant matrix A. If $r_1 > r_2 > 0$ then the equilibrium for the system $\vec{w}' = A\vec{w}$ is

$\rightarrow A$. an unstable node.
B. a stable node.
C. a saddle point.
D. none of the above.

---

4. Suppose r_1, r_2 are the eigenvalues for a 2×2 constant matrix A. If $r_1 < r_2 < 0$ then the equilibrium for the system $\vec{w}' = A\vec{w}$ is

A. an unstable node.
$\rightarrow B$. a stable node.
C. a saddle point.
D. none of the above.

---

5. Suppose A is a 2×2 constant matrix. If the equilibrium for the system $\vec{w}' = A\vec{w}$ is a spiral point then the eigenvalues are

A. positive.
B. negative.
$\rightarrow C$. complex.
D. none of the above.

Quiz 6

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1. Suppose A is a 2×2 constant matrix, and that the eigenvalues are complex with positive real part. Which way do the trajectories for the system $\vec{w}' = A\vec{w}$ spiral?

A. Clockwise.
B. Counter-clockwise.
$\rightarrow C$. Cannot be determined from this information.

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2. Suppose A is a 2×2 constant matrix, and that the eigenvalues are complex with positive real part. Which way do the trajectories for the system $\vec{w}' = A\vec{w}$ spiral?

A. Into the origin.
$\rightarrow B$. Away from the origin.
C. Cannot be determined from this information.

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3. Suppose A is a 2×2 constant matrix, with eigenvectors $\vec{\xi}_1$ and $\vec{\xi}_2$, and corresponding eigenvalues r_1 and r_2. If $r_1 > r_2 > 0$ then as $t \to +\infty$ almost every solution to the system $\vec{w}' = A\vec{w}$ is asymptotic to which direction?

$\rightarrow A$. $\vec{\xi}_1$.
B. $\vec{\xi}_2$.

---

4. Suppose A is a 2×2 constant matrix, with eigenvectors $\vec{\xi}_1$ and $\vec{\xi}_2$, and corresponding eigenvalues r_1 and r_2. If $r_1 < r_2 < 0$ then as $t \to +\infty$ almost every solution to the system $\vec{w}' = A\vec{w}$ is asymptotic to which direction?

A. $\vec{\xi}_1$.
$\rightarrow B$. $\vec{\xi}_2$.

---

5. Suppose A is a 2×2 constant matrix, with positive trace and negative determinant. What is the equilibrium for the system $\vec{w}' = A\vec{w}$?

$\rightarrow A$. A saddle point.
B. An unstable spiral.
C. An unstable node.
D. None of the above.

Quiz 7

1. What does "reduction of order" reduce $ay'' + by' + cy = g$ to?
→*A*. A pair of first-order linear equations.
B. A pair of first-order separable equations.
C. A pair of first-order homogeneous equations.
D. None of the above.

2. What is the form of the solution used in "variation of parameters"?
A. $c_1y_1 + c_2y_2$, where y_1 and y_2 are solutions of the homogeneous equation and c_1 and c_2 are constants.
B. c_1y_1, where y_1 is a solution of the homogeneous equation and c_1 is constant.
C. c_1y_1, where y_1 is a solution of the nonhomogeneous equation and c_1 is constant.
→*D*. None of the above.

3. What is the subtraction formula for cosine?
→*A*. $\cos(A - B) = \cos(A)\cos(B) + \sin(B)\sin(A)$.
B. $\cos(A - B) = \cos(A)\sin(B) + \cos(B)\sin(A)$.
C. $\cos(A - B) = \cos(A)\sin(B) - \cos(B)\sin(A)$.
D. None of the above.

4. Suppose $y = 3.1\cos(\sqrt{2}t + 3)$. What is the phase shift of y?
A. $3\sqrt{2}$.
B. $3/\sqrt{2}$.
→*C*. $-3/\sqrt{2}$.
D. None of the above.

5. Suppose $y = 3.1\cos(\sqrt{2}t + 3)$. What is the period of y?
A. 3.1.
B. $2\sqrt{2}\pi$.
→*C*. $\sqrt{2}\pi$.
D. None of the above.

Quiz 8

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1. The series $\sum \frac{1}{n}$
→*A*. diverges.
B. converges.
C. has radius of convergence equal to 1.
D. None of the above.

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2. The series $\sum (x-1)^n/n$
A. defines an analytic function when $|x-1|<1$.
B. diverges when $|x-1|>1$.
C. has radius of convergence equal to 1.
→*D*. All of the above.
E. None of the above.

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3. The series $\sum x^n/n!$
→*A*. defines an analytic function for all x.
B. diverges when $|x|>1$.
C. has radius of convergence equal to 1.
D. All of the above.
E. None of the above.

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4. The series $\sum \binom{\alpha}{n} x^n$
A. defines an analytic function for all x.
→*B*. diverges when $|x|>1$.
C. has radius of convergence equal to ∞.
D. All of the above.
E. None of the above.

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5. $\displaystyle\int_0^t \frac{ds}{1+s^2} =$
A. $1-s^2+s^4-s^6+\cdots$, for all t.
B. $1+t+\frac{1}{2}t^2+\frac{1}{3!}t^3+\cdots$, when $|t|<1$.
→*C*. $t-\frac{1}{3}t^3+\frac{1}{5}t^5-\frac{1}{7}t^7\cdots$, when $|t|<1$.
D. None of the above.

Quiz 9

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1. What is $\mathcal{L}\left\{t^4\right\}$?
A. s^4
B. s^{-4}
C. s^{-5}
→*D.* None of the above.

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2. What is $\Gamma(5)$?
→*A.* 24
B. 120
C. 720
D. None of the above.

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3. What is $\mathcal{L}\{1\}$?
A. 1
B. s
→*C.* $1/s$
D. None of the above.

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4. What is $\mathcal{L}\{\cos(3t)\}$?
A. $3/(s^2+9)$
→*B.* $s/(s^2+9)$
C. $s/(s^2+3)$
D. None of the above.

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5. Suppose $Y(s) = \mathcal{L}\{y(t)\}$ and $y'(0) = 3$. What is $\mathcal{L}\{y'(t)\}$?
A. $Y'(s) - 3$
→*B.* $sY(s) - 3$
C. $sY'(s) - 3$
D. None of the above.

Quiz 10

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1. Consider a system $ay'' + by' + cy = 0$, where a, b, c are all positive constants. If the discriminant is positive then the system is

A. underdamped.
→*B*. overdamped.
C. critically damped.
D. None of the above.

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2. Consider a system $ay'' + cy = g$, where a and c are positive constants. The system exhibits beats when

A. $g = F_0 \cos(\omega t)$, where ω equals the natural frequency.
B. $g = F_0 t \cos(\omega t)$, where ω equals the natural frequency.
→*C*. $g = F_0 \cos(\omega t)$, where ω does not equal the natural frequency.
D. None of the above.

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3. Consider a system $ay'' + cy = g$, where a and c are positive constants. The system exhibits resonance when

→*A*. $g = F_0 \cos(\omega t)$, where ω equals the natural frequency.
B. $g = F_0 t \cos(\omega t)$, where ω equals the natural frequency.
C. $g = F_0 \cos(\omega t)$, where ω does not equal the natural frequency.
D. None of the above.

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4. Consider a system $ay'' + by' + cy = F_0 \cos(\omega t)$, where a, b, c are all positive constants. The transient solution is the same as

→*A*. the homogeneous solution.
B. the driving function.
C. the forced response.
D. None of the above.

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5. Consider a system $ay'' + by' + cy = F_0 \cos(\omega t)$, where a, b, c are all positive constants. The steady-state solution is the same as

A. the homogeneous solution.
B. the driving function.
→*C*. the forced response.
D. None of the above.