Course Outline

Math 4350/5350/7350, Spring 02

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Introduction

Welcome to Applied Linear Algebra! This is a subject with lots of subtle, interesting problems whose solutions miraculously turn out to be widely applicable, in nearly every corner of science and engineering. The central theme is explained in the appendix to our text: "Numerical analysis studies algorithms used to solve problems of continuous mathematics." Numerical linear algebra is the most important example of numerical analysis. Your first assignment is to read the appendix.

The key word in Trefethen's definition is "algorithm". An algorithm is a step-by-step procedure, a recipe for solving a problem. Algorithms are the abstract structures that underly computer programs. We will be interested in the *stability* of these algorithms. That is, we will want to understand how the output of the algorithm is affected by small changes in the input.

To fully understand this we need to separate out a distinct concept: conditioning. We need to know whether a problem is *well-conditioned* or *ill-conditioned*. A well-conditioned problem is one where small changes in the data for a problem imply small changes in the solution to that problem. Note the subtle distinction here. Even when we use a perfect algorithm to solve an ill-posed problem, we expect to get wildly varying results when we vary the input. On the other hand if we use an unstable algorithm to solve a well-posed problem, we cannot expect to get anything near the correct answer if we make small changes to the input data.

The foundation for all of our algorithms are various matrix factorizations. An example of a matrix factorization that should be familiar to you is the LU factorization that arizes in Gaussian elimination. It turns out that this algorithm is quite atypical — its numerical effectiveness is especially nasty to understand. In this course we will address Gaussian elimination and LU factorization only after thoroughly studying more typical and, it turns out, more fundamental matrix factorizations: the singular value decomposition and the Gram-Schmidt (QR) factorization are the most important for us..

We will approach new concepts largely thru examples. This means we will be looking at the results of computer experiments to see how things work in practice. The best way to understand what algorithms do is to implement them on a computer and run some experiments. We will not be doing any programming nor any experiments in class, however.

Calendar

Review lectures	Wednesday	16 January
	Friday	18 January
ML King Day (no classes)	Monday	21 January
Last Day to Add/Drop	Friday	28 January
Last Day to Withdraw	Friday	8 March
Spring Break		11–15 March
Exam 1:	Wednesday	20 March
Chapters I, II, III	6–8pm	UH 3008
Last Day of Classes	Friday	3 May
Exam 2:	Monday	6 May
Chapters IV, V	8–10am	ST 118

Text and syllabus

We will cover the material in chapters I–V of *Numerical Linear Algebra*, by L Trefethen and D Bau. This book is a wonderful departure from other texts on numerical analysis and linear algebra, which can be very dry and uninspiring. Trefethen developed this book over several yeas from lecture notes in his highly successful course at Cornell. (David Bau, was a student in Trefethen's course.) It is a fun book, and I think you will enjoy it.

Whether or not you enjoy it, I certainly expect you to read this book. The concepts are subtle and will demand some real thinking, but this is a very readable book. If you spend time studying its examples in detail then you will get a lot of insight into the fascinating subject of numerical linear algebra. Note that this schedule implies an average pace of two sections (the text calls them"lectures") per week. Please make every effort to keep up this pace in your reading.

Prerequisites

This is *not* a first course in linear algebra. I assume that you are thoroughly familiar with systems of equations and Gaussian elimination. I assume that you have already learned about bases and linear independence, vector spaces and linear transformations, rank and nullity, matrix representation and change of basis, and eigenvalues and eigenvectors. This is prerequisite material and if you never learned these topics then almost certainly you will not be able to complete this course successfully.

However, it is quite possible that you learned this material a long time ago, and need some review. So, I will give supplemental lectures before class on Wednesday and Friday of the first week, assign review problems in these lectures. These supplementary sessions are wholly optional, but if you think you might need help to refresh your understanding of this prerequisite material, I strongly recommend that you attend these sessions and work on the review problems. Whether or not you attend these sessions, you should spend the first week reviewing the fundamentals of linear algebra. If by the end of a week you find that you are still not comfortable with the prerequisites then you should seriously reconsider your enrollment in this course. If the prerequisite material is difficult, the material of this class will be impossible.

Programming

I want to make it clear that you do not need to know any computer programming in this course. Quite naturally we will be using simple computer programs to experiment with the main ideas we cover, but you will not be required to write any computer programs. Grad students will be expected to complete two extended assignments: either writing python programs that implement the algorithms we discuss in class, or else using python programs that I have written and analyzing the data. It is your choice. These two projects will determine 20% of your final grade.

Assignments and quizzes

Once or twice per week, perhaps more often, we will begin work in class on a problem from the text, and then you will complete this problem at home and turn your solution in for grade the following class. In your solutions I expect neat work, showing all details, accompanied by written explanations, in complete sentences.

Each of these assignments will consist of exactly one problem from the text, but I will also assign other, similar problems from the text, to practice at home. You will not turn these in for grade, but you should work on all of the problems since I will draw exam questions from the assigned problems.

Also once or twice per week we will have a short quiz, with two or three questions, worth 1 point each. Periodically I will give you a list of questions to practice at home, and on quiz days (which will *not* be announced ahead of time) I will choose a couple from the current list. Typically the quiz problems will cover things like definitions, terminology, important theorems, and so forth. Some questions may involve short computations; some may be true/false or multiple-choice questions; some may ask you for examples or explanations.

Exams

We will have two 2-hour exams, each worth 100 points. The exam questions will be taken directly from the practice questions I give you in class. This includes all practice quiz questions, both those we use in class and those we do not use in class, and all practice problems from the text, both those that are collected for grade and those not collected. Please note that exam 1 is scheduled in the evening (see the calendar). If you cannot take the exam at the scheduled time you must make arrangements with me as soon as possible.

Attendance

I will not give make-up quizzes nor allow assignments to be turned in late, under any circumstances. If you miss class on a quiz day then you miss a couple of points. This will not affect your final grade if you miss class once or twice per semester. However, if you are in the habit of missing class regularly then history shows that you will probably fail, and that making up missed quizzes will not help. Thus, I will save both of us a lot of hassle by sticking to the no make-up rule. There will be no exceptions. Don't ask.

I will give make-up exams only in case of a documented emergency, such as illness or a funeral. If you are sick the day of the exam then you must call that same day if you expect to be able to make up the exam. If I am not in my office then you can leave a voice mail message. If you have a funeral or other emergency then you must arrange for a make-up exam ahead of time. If you fail to show up for an exam and do not contact me about it until afterwords then you will not be able to make up that exam. You will get a 0 for that exam.

Grades

I will determine final grades based on the class-wide distribution of points earned. Exams contribute 200 possible points, quizzes and assignments contribute approximately 100 possible points altogether. I want to emphasize that you are not in direct competition with each other. I do not feel obligated to given any grades of F, or any grades of A for that matter. I will not split hairs. I do not have a set grade scale. You will find that the grade distribution breaks into obvious groups. Historically in my classes it takes 85–90% of the points to earn an A; around 70–75% for a B; and around 60% for a C. However, these are not rigid targets, just historical observations. After the first exam I will post a histogram of total points earned, and this should give you a clearer idea of where you stand.

For the grad students your total points will be a weighted average, with exams, quizzes, and practice problems contributing 80%, the two projects contributing 20%.

If you want me to post your grades under a nickname, bring me a 3×5 card with your name, an email address, and the nickname you wnat to use — preferably something not obvious.