# Math 2850-021, Summer 2014

## Quiz 1

1. Find the angle between the velocity and acceleration vectors at time $t = 0$ of the particle whose position is given at time $t$ by

$$\mathbf{r}(t) = \frac{\sqrt{2}}{2}t\mathbf{i} + (\frac{\sqrt{2}}{2}t - 16t^2)\mathbf{j}.$$

2. A particle travels with constant acceleration $2\mathbf{i} + \mathbf{j} + \mathbf{k}$. At time $t = 0$ the particle is located at the point $(1, -2, 2)$ and has velocity $-2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. Find its position vector $\mathbf{r}(t)$ at time $t$.

## Quiz 2

1. For the indicated parametric curve find the unit tangent vector at time $t$.

$$\mathbf{r}(t) = 3\sin(2t)\,\mathbf{i} + 3\cos(2t)\,\mathbf{j} + 8t\,\mathbf{k}.$$

2. Find the length of the indicated parametric curve.

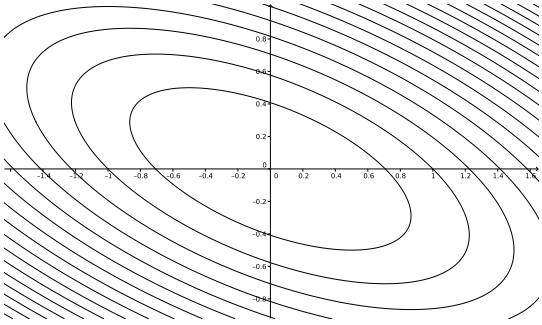
$$\mathbf{r}(t) = 3\sin(2t)\,\mathbf{i} + 3\cos(2t)\,\mathbf{j} + 8t\,\mathbf{k},\ 0 \le t \le \pi.$$

## Quiz 3

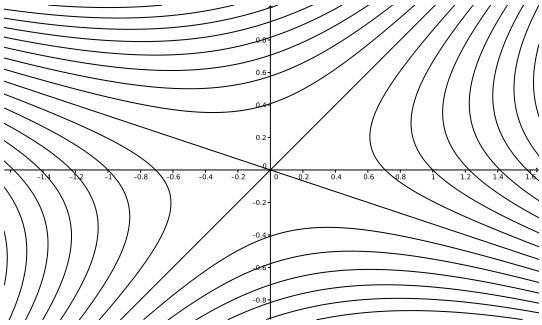
1. Which of the graphs below is a contour map of the function $f(x, y) = x^2 + 3y^2 + 2xy$?

2. Which of the graphs below is a contour map of the function $f(x, y) = \dfrac{1}{xy}$?

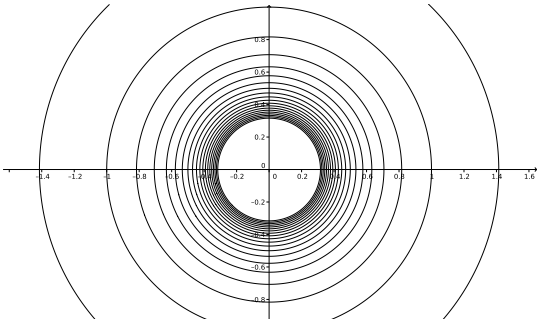
*A*.



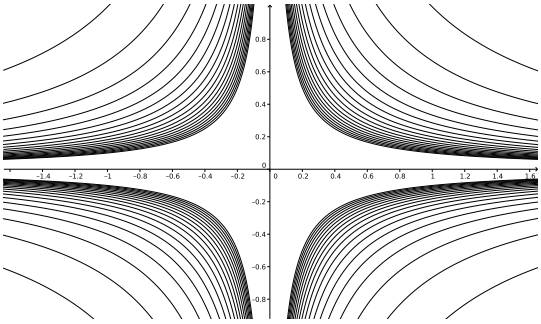
*B*.



*C*.



*D*.



## Quiz 4

1. For $f(x, y, z) = yz\ \ln(xy)$ compute $f_x$, $f_y$, and $f_z$.
2. For $w = x^2 + y^2$, $x = t^3$, $y = t^5$ compute $d(w(x(t), y(t))/dt$ using the multivariable Chain Rule.

## Quiz 5

1. For $f(x, y, z) = e^x \cos(yz)$, $P_0 = (0, 0, 0)$, and $\mathbf{u} = \langle 2, 1, -2 \rangle$ compute the derivative of $f$ at $P_0$ in the direction of $\mathbf{u}$.

2. Find an equation to the tangent plane of the surface of $e^x \cos(yz) = 1$ at the point $(0, 0, 0)$.

## Quiz 6

1. Find the absolute maximum of $T(x, y) = x^2 + xy + y^2 - 6x$ on the closed rectangular plate bounded by the lines $x = 0$, $x = 5$, $y = -3$, and $y = 0$.

2. Find the shortest distance from the origin to a point on the curve $x^2 y = 2$.

## Quiz 7

1. Evaluate $\displaystyle\iint_R \frac{\sqrt{x}}{y}\, dA$ where $R = \{(x, y) \mid 0 \le x \le 4,\ 1 \le y \le 2\}$.

2. Reverse the order of integration for $\displaystyle\int_0^{\ln 2} \int_{e^y}^{2} (x + y)\, dx\, dy$.

## Quiz 8

1. Change $\displaystyle\int_0^2 \int_0^x y\, dy\, dx$ to polar coordinates.

2. Evaluate $\displaystyle\int_0^1 \int_y^1 \frac{e^x\, dx\, dy}{x}$.

## Quiz 9

1. Find the centroid of the region between the $x$-axis and the curve $y = \cos x$ for $0 \le x \le \pi/2$.

2. Find the moment of inertia about the about the $y$-axis of a thin rectangular plate cut from the first quadrant by the lines $x = 2$ and $y = 1$ if the density $\delta(x, y) = x + y + 1$.

## Quiz 10

1. Find the volume of the right circular cylinder whose base is the circle $r = 3$ and whose top lies on the plane $z = 5 - x$.

2. Find the volume of the solid that lies below the $xy$-plane and between the sphere $\rho = 1$ and the cardioid of revolution $\rho = 1 + \cos\phi$.

## Quiz 11

1. Find the line integral of $f(x, y, z) = x^2 + yz$ over the line segment from $(0, -1, 1)$ to $(-1, 0, 2)$.

2. Find the work done by the force field $\mathbf{F} = \langle -y, x \rangle$ on a particle moving counterclockwise one full turn around the circle $x^2 + y^2 = 1$.

## Quiz 12

1. Suppose that $\mathbf{F} = \langle M, N \rangle$ is a smooth vector field in some planar domain $R$. Which of the following are true statements? Justify your answers.

   *A.* If $\mathbf{F}$ is conservative then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every piecewise smooth closed path $C$ in $R$.

   *B.* If $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every piecewise smooth closed path $C$ in $R$ then $\mathbf{F}$ has a potential.

   *C.* If $M_y = N_x$ then $\mathbf{F}$ is conservative.

   *D.* If $\mathbf{F}$ has a potential then $M_y = N_x$.

2. Which of the following are conservative fields? For those that are conservative find a potential. For those that are not explain why not.

   *A.* $\mathbf{F} = \langle x, y \rangle$

   *B.* $\mathbf{F} = \langle y, x \rangle$

   *C.* $\mathbf{F} = \langle -y, x \rangle$

   *D.* $\mathbf{F} = \langle -y/(x^2 + y^2), x/(x^2 + y^2) \rangle$

## Quiz 13

1. Let $\mathbf{F} = \langle x^3 - 3xy^2 + 2x, -3x^2y + y^3 + 2y \rangle$ and $C$ the boundary of the upper half of the unit disk. Compute $\oint_C \mathbf{F} \cdot \mathbf{T}\, ds$.

2. Let $\mathbf{F}$ and $C$ be as in problem 1. Compute $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds$.

## Quiz 14

1. What is the area element for the surface $3x - 2y + 5z = 30$?

2. What is the area element for the surface $x^2 + z^2 = 9$?

## Quiz 15

1. Evaluate $\iint_S z\,d\sigma$, where $S$ is the portion of the surface $3x + 2y + 5z = 30$ that lies in the first octant.
2. Evaluate $\iint_S x^2\,d\sigma$, where $S$ is the portion of the surface $x^2 + z^2 = 9$ that lies between the planes $y = 0$ and $y = 2$.

## Quiz 16

1. Evaluate $\int_C \nabla f \cdot d\mathbf{r}$, where $f(x, y, z) = xy^2 + 2z^3$ and $C$ is the path that begins at the point $(0, 1, 1)$, traverses the line segment to $(2, 0, 0)$, then turns and traverses the line segment to $(2, 0, -1)$.
2. Let $\mathbf{F} = \langle x^2 - y^2 + 2x, -2xy + 5y\rangle$ and $C$ the boundary of the upper half of the unit disk. Compute $\oint_C \mathbf{F} \cdot \mathbf{n}\,ds$.
3. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle y, xz, x^2\rangle$ and $C$ is the boundary of the triangle cut from the first octant by the plane $x + y + 2z = 2$, oriented counterclockwise when viewed from above.
4. Evaluate $\iint_S \mathbf{F} \cdot \mathbf{n}\,d\sigma$, where $\mathbf{F} = \langle x^2, -2xy, 3xz\rangle$ and $S$ is the boundary of the wedge cut from the first octant by the sphere $x^2 + y^2 + z^2 = 4$ .