Miscellaneous Problems

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Here I list problems related to some of the previous applications. I will probably add a few more in the near future.

1. Fourier theory is an example of least squares approximation. In this problem we apply least squares approximation in another context: fitting a polynomial to observed data. You will present your findings to class.

   - Suppose we make several measurements (x_i, y_i) — say, mass and electrical charge on an oil droplet — and we wish to discover the relationship between x and y. We can try and fit a polynomial curve $y = p_n(x) = a_n x^n + \cdots + a_0$. Write down the requirements $p_n(x_i) = y_i$ as a system of linear equations. In general, if n is small then this is an overdetermined system
   - Generate test data as follows: start with a polynomial $q(x)$ of moderate degree (say 3 or 4) and choose several values for the x_i (perhaps 10 altogether) then compute $y_i = q(x_i) + r_i$, where r_i is a small random value. Take $n = 0, 1, 2, \ldots, 12$, and form the corresponding matrix (as above) for the $p_n(x)$. Use QR-factorization on each of these matrices to find the best fitting $p_n(x)$.
   - Plot your data together with each of the polynomials. Compute the residuals — the differences between the data points and the values of the polynomial model. Which n gives the fit that is best? Explain your choice.

2. If we build a projective plane using another number system — for example a finite field — then we obtain a "geometry" that does not match our visual field, but which nonetheless has important applications, notably for statistical analysis and the design of experiments. In this problem you will look at a few simple examples and present your results to the class.

   - One way to visualize a finite projective plane is using a graph. For each of the fields of 2, 3, 4, and 5 elements, draw a picture of the projective plane using both the points and lines as vertices, and joining a point-vertex to a line-vertex when the point lies on the line.
   - A projective plane can be characterized by the following remarkable properties: (A) Any two points lie together on a unique line. (B) Any two lines intersect in a unique point. Verify these two properties for each of your finite projective planes.
   - Illustrate what 3×3 linear transformations do to the projective planes built from the fields with 2 and 3 elements.