Computer Graphics

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April 17, 2002

In computer representations of 3-dimensional objects, the information on the object is represented internally in 3 dimensions, and then it is projected onto a plane, which represents the screen. t is convenient to let the first two coordinates represent the dimensions of the screen, and the third represent the orthogonal dimension of "depth". There are are several ways to project the object. For instance the third coordinate could simply be ignored. A second way is to project towards a point of view of the computer user.

Problems

If there is sufficient interest in this application, then I will post more problems.

1. Imagine you are part of a design team for a computer graphics application, and your task is to write "callback" functions which manipulate the image on the screen based on mouse input from the user. Your team is in prototyping mode so you are to write your code in the language python (available at `http://python.org`). Moreover you are to assume that the object being displayed is a tetrahedron, stored internally as a triple of triples — that is, 3 points in virtual 3-space. Mouse input will come in the form of a pair of pairs — that is, 2 points on the screen.

   - The first function you need to write is a *dilatation* (rescaling). In this, imagine that the 2 points come from a click and drag operation: the user clicks at one point of the tetrahedron, and drags it to another. Write a python function whose output is a function which computes the matching dilation, computed with respect to the tetrahedrons center of mass.
   - The second function you are to write computes a translation. Again, the mouse input comes from a click and drag.
   - The third function you are to write computes a rotation. Again the mouse input comes from click and drag, but this time you are to interpret the data as determining a velocity vector: the tetrahedron is to rotate about an axis passing thru its center of mass, perpendicular to the velocity vector. The tangential speed of rotation should be the length of the input velocity. Note that the function which is output in this case will have one extra argument: time.
   - (Extra credit.) Use a gui toolkit (Tk or Java's swing, for example) to illustrate your functions.

2. In this second problem we are going to explore projection from a point. In this projection, we take the origin to be the viewer's eye. We will call the plane of the view screen the *projective plane*. Each line of sight — that is, each line in 3-space passing thru the origin — corresponds to a point on the projective plane. Each line on the projective plane corresponds to a plane thru the origin in 3-space — the plane swept out by the viewer's eye as he scans the line on the projective plane. Similarly a circle on the projective plane corresponds to a cone in 3-space (with vertex at the origin).

   In this problem you will investigate what happens to conic sections in the projective plane under the influence of linear transformations of 3-space.

   - If (x, y) are the coordinates of a point on the projective plane, then what are the coordinates of the points on the corresponding line in 3-space? The 3-dimensional coordinates are called the *homogeneous coordinates* of the point. Under what conditions do homogeneous coordinates (t_1, t_2, t_3) and (u_1, u_2, u_3) determine the same point on the projective plane?
   - Observe that there are "points at infinity" on the projective plane, corresponding to lines in 3-space that are parallel to the projective plane (and which pass thru the origin). What are the homogeneous coordinates of the points at infinity?
   - Which linear transformations keep all of the points at infinity out at infinity?
   - If a linear transformation A fixes a point P on the projective plane, then what is the relationship of A and the homogeneous coordinates of P?
   - Let $ax + by = c$ be the equation of a line on the projective plane. Find an equation for the homogeneous coordinates. Do the same for the equation of a circle on the projective plane.
   - Explore the possibilities when you start with a circle on the projective plane, apply a linear transformation of its homogeneous coordinates, and view the result on the projective plane. What geometric figures can arise this way?