

Applied Linear Algebra Sample Exam

1. Give a quiz-like answer to the following questions.
 - a. Express matrix multiplication in terms of columns.
 - b. What is the outer product and what is its significance?
 - c. What is the representation of a linear transformation?
2. Give a quiz-like answer to the following questions.
 - a. What is the rank of a matrix? How can you determine the rank?
 - b. List the bilinearity rules?
 - c. What does it mean for a matrix to be non-singular?
3. Let A be an $m \times n$ matrix and let B be a submatrix of A , that is, a $u \times v$ matrix ($u = , v = n$) obtained by selecting certain rows and columns of A .
 - a. Explain how B can be obtained by multiplying A by certain row and column "deletion matrices."
 - b. Using this product, show that $\|B\|_p \leq \|A\|_p$ for any p with $1 \leq p \leq \infty$.
4. Prove the following: The rank of a matrix A is the number of non-zero singular values of A .
5. If P is an orthogonal projector then $I-2P$ is unitary. Give a geometric interpretation of $I-2P$. Do the same with $2P-I$.
6. Describe the full and reduced QR factorization and explain how they are different.
7. Using the provided Classical and Modified Gram-Schmidt algorithms step through and carefully list each step of both algorithms with the provided matrix. Compare the two algorithms and the results of each.
8. The Householder algorithm is considered an Orthogonal Triangularization where the Gram-Schmidt is a Triangular Orthogonalization. Explain the difference in these two terms and describe the differences in the order of outcomes.