Math 2850-005, Fall 2014

Quiz 1

1. Find d^2y/dx^2 at the point on the parametric curve $x = t + e^t$, $y = 1 - e^t$ where $t = 0$.

2. Find the area *inside* the cardioid $r = 1 + \cos\theta$ and *outside* the cardioid $r = 1 - \cos\theta$.

Quiz 2

1. Find the equation of the plane that contains the point $(-1, 2, 0)$ and is perpendicular to the parametric line $\mathbf{r}(t) = \langle 2t + 1, 3 - t, t - 2\rangle$.

2. Find the distance from the point $(2, 2, 3)$ to the plane $2x + y + 2z = 4$.

Quiz 3

1. For the particle whose position at time t is $\mathbf{r}(t) = \langle e^{-3t}, 2\sin(2t), 2\cos(2t)\rangle$ find the speed and direction of motion when $t = 0$.

2. For the curve in problem 1 find $\cos(\theta)$, where θ is the angle between the velocity and acceleration vectors when $t = 0$.

Quiz 4

1. Let $\mathbf{r} = \langle \cos(t) + t\ \sin(t), \sin(t) - t\ \cos(t)\rangle$ for $t > 0$. Find $\mathbf{T}$.

2. For the curve in problem 1 find κ.

Quiz 5

1. Let $\mathbf{r} = \langle 2t, t^2, t^3/3\rangle$. Find a_T and a_N.

2. For the curve in problem 1 find κ.

Quiz 6

What are the logical implications between the following statements?

A. The force exerted by the Sun on a planet lies along the line joining the Sun to the planet.

B. The magnitude of the force exerted by the Sun on a planet is inversely proportional to the distance from the Sun to the planet.

C. The angular momentum of a planet as it orbits the Sun is constant.

D. The line from the Sun to a planet sweeps out equal areas in equal time intervals.

Quiz 7

1. Let R be the domain of the function $f(x, y) = \sqrt{1 - x^2 - y^2}$. Mark each of the following True or False. *Justify each of your responses!*

   A. R is bounded.

   B. R is open.

   C. R is closed.

2. Let R be the domain of the function $f(x, y) = \ln(xy)$. Mark each of the following True or False. *Justify each of your responses!*

   A. R is bounded.

   B. R is open.

   C. R is closed.

Quiz 8

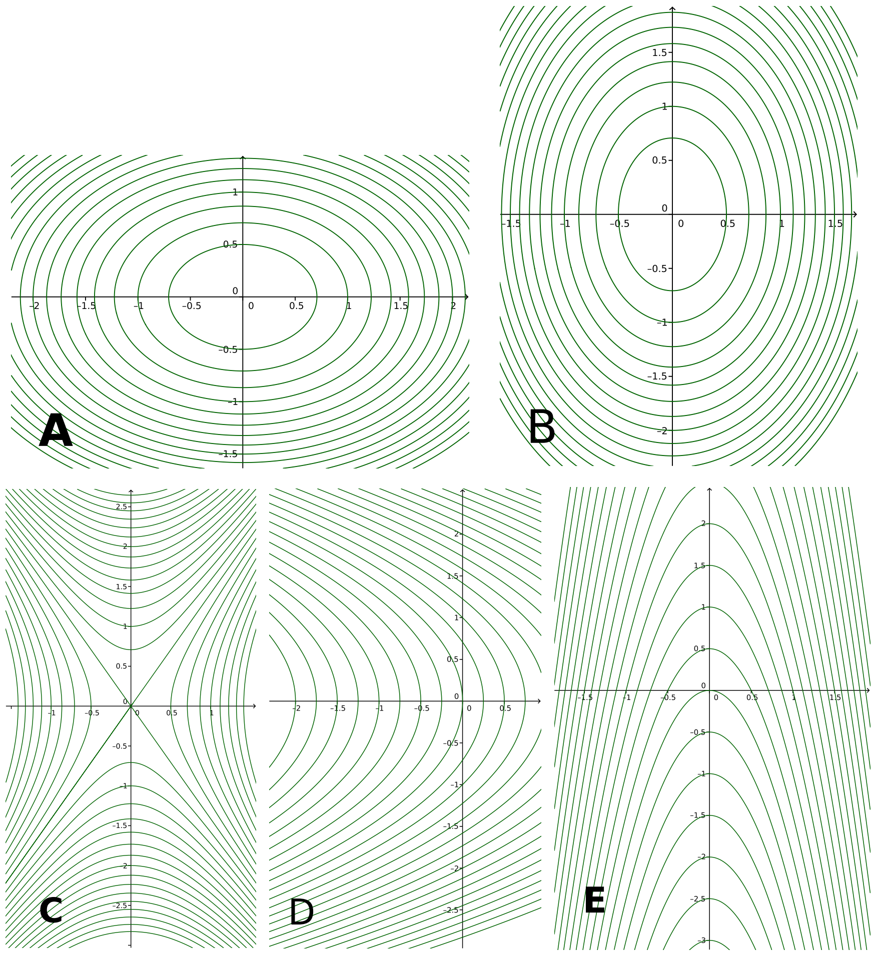
1. Compute $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$, or else show that the limit does not exist.

2. Compute $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^2}$, or else show that the limit does not exist.

Quiz 9

Match each of the following functions to its contour plot of level curves below. *Justify each of your responses!*

1. $f(x, y) = 4 - 2x - y^2$
2. $f(x, y) = 4 - 2x^2 - y$
3. $f(x, y) = 4 - 2x^2 - y^2$
4. $f(x, y) = 4 + 2x^2 - y^2$



Quiz 10

1. Compute f_x, where $f(x, y, z) = ye^{xz} + \sin(y^2 z)$
2. Compute f_{xz}, where $f(x, y, z)$ is the function above.

Quiz 11

1. Find the values of $\partial z/\partial x$ and $\partial z/\partial y$ at the point $(1, 1, 1)$ on the surface $z^3 - xy + yz + y^3 = 2$.
2. Find the derivative of the function $f(x, y) = 2xy - 3y^2$ at the point $(1, 1)$ in the direction of $\langle 4, 3\rangle$.

Quiz 12

1. What is the direction in which the function $f(x, y, z) = xy^2 + 3yz$ increases most rapidly at the point $(1, 1, 0)$?
2. What is the linearization of the function above at the same point $(1, 1, 0)$?

Quiz 13

Find and classify the critical points of the function $f(x, y) = x^3 + 3xy + y^3$.

Quiz 14

What are the maximum and minimum values of $f(x, y) = 5x - 12y$ on the circle $x^2 + y^2 = 1$, and where do these occur?

Quiz 15

1. Evaluate $\displaystyle\iint_R xye^{xy^2}\, dA$, where R is the rectangle $[0, 2] \times [0, 1]$.
2. Evaluate $\displaystyle\iint_R \frac{y}{xy + 1}\, dA$, where R is the rectangle $[0, 1] \times [0, 3]$.

Quiz 16

1. Evaluate $\displaystyle\int_0^1 \int_y^1 x^2 e^{xy}\, dx\, dy$.
2. Evaluate $\displaystyle\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{dy\, dx}{y^4 + 1}$.

Quiz 17

Convert $\displaystyle\int_0^1 \int_x^{\sqrt{2-x^2}} (x + 2y)\, dy\, dx$ to polar coordinates, and then evaluate the integral.

Quiz 18

1. Compute the average value of the function $f(x, y) = xy$ over the portion of the unit disk $x^2 + y^2 \leq 1$ that lies in the first quadrant.
2. Evaluate $\displaystyle\int_0^2 \int_0^{2-x} \int_0^{2-x-y} dz\, dy\, dx$.

Quiz 19

Let T be the tetrahedron bounded by the coordinate planes and the plane $x + y + z = 2$. Suppose that T has density $\delta = 3 - z$. Compute the center of mass of T.

Quiz 20

1. Let R be the thin plate in the xy-plane bounded by curves $x = y^2$ and $x = 2y - y^2$. Suppose that R has constant density $\delta = 2$. Compute the moments of inertia of R about the x-axis, about the y-axis, and about the origin.

2. Suppose that the thin plate described above has been milled to a nonconstant density $\delta = xy + 1$. Compute the center of mass of this milled plate.

Quiz 21

Rewrite the integral $\iint_R (2x^2 - xy - y^2)\, dA$ in terms of u and v, where $u = x - y$, $v = 2x + y$, and R is the region bounded by the lines $y = -2x + 4$, $y = -2x + 7$, $y = x - 2$, and $y = x + 1$. *Extra credit:* Evaluate the integral in both coordinate systems.

Quiz 22

1. Evaluate $\int_C f\, ds$ where $f(x, y) = x + y^2$ and C is the arc of the circle $x^2 + y^2 = 8$ from $(2, -2)$ to $(2, 2)$.

2. Find the center of mass of the piece of wire that lies along the curve $y = x^2$ from $(0, 0)$ to $(2, 4)$ and has density $\delta = 1 + xy$.

Quiz 23

1. Evaluate $\int_C f\, ds$ where $f(x, y, z) = xy + z^2$ and C is the straight line segment from $(1, -1, 3)$ to $(0, 4, 4)$.

2. Evaluate $\int_C f\, ds$ where $f(x, y) = xy + 3$ and C is the unit circle, oriented counterclockwise.

Quiz 24

1. Evaluate $\int_C \mathbf{F} \cdot \mathbf{T}\, ds$ where $\mathbf{F}(x, y) = \langle yx, x - y \rangle$ and C is the circle $(x-2)^2 + y^2 = 4$, oriented clockwise.

2. Evaluate $\int_C yz\, dx - 3xz\, dy + 2x^2\, dz$ where C consists of the line segment from $(0, 2, -1)$ to $(3, 2, 0)$ together with the line segment from $(3, 2, 0)$ to $(1, 3, 0)$.

Quiz 25

1. Find the circulation of the vector field $\mathbf{F}_1(x, y) = \langle -y, x \rangle$ around the unit circle, oriented counterclockwise.

2. Find the flux of the vector field $\mathbf{F}_2(x, y) = \langle x, y \rangle$ across the unit circle, oriented counterclockwise.

3. *Extra credit:* Are either of the above fields $\mathbf{F}_1$ or $\mathbf{F}_2$ conservative? Explain!

Quiz 26

1. Find the flux of the vector field $\mathbf{F}(x, y) = \langle 2xy, x - y^2 \rangle$ across the boundary of the *right half* of the disk $x^2 + y^2 \le 2$, oriented counterclockwise.

2. Find the circulation of the vector field $\mathbf{F}(x, y) = \langle 3y^2, 2xy \rangle$ around the triangle with vertices $(1, -1)$, $(0, 2)$, and $(-3, 0)$, oriented counterclockwise.

Quiz 27

1. Find a potential function for the vector field $\mathbf{F}(x, y, z) = \langle x/\rho^3, y/\rho^3, z/\rho^3 \rangle$, where $\rho^2 = x^2 + y^2 + z^2$.

2. Explain why there is no potential function for the vector field $\mathbf{F}(x, y) = \langle -y/r^2, x/r^2 \rangle$, where $r^2 = x^2 + y^2$.

Quiz 28

1. Use Green's Theorem to determine both the outward flux of the vector field $\mathbf{F} = \langle \sin(y) + 2x, 3y - \cos(x) \rangle$ across the triangle with vertices $(0, 2)$, $(0, 0)$, and $(2, 0)$ and also the counterclockwise circulation around this triangle.

2. Use Green's Theorem to determine both the counterclockwise circulation of the vector field $\mathbf{F} = \langle xy^2 + 2y, 5x + x^2y \rangle$ around the circle of radius 2 centered at $(2, 0)$ and also the outward flux across this circle.

Quiz 29

1. Use Green's Theorem to determine the counterclockwise circulation of the vector field $\mathbf{F} = \langle 3x - y, 2x + y \rangle$ around the circle of radius 2 centered at the origin.

2. Use Green's Theorem to determine the outward flux of the vector field $\mathbf{F} = \langle x^2 + 5y, xy + 2x \rangle$ across the triangle with vertices $(0, 2)$, $(0, 0)$, and $(2, 0)$.

Quiz 30

1. Let $\mathbf{F} = \langle 2xy - z^2, yz + 3x^2, 2y^2 - xz \rangle$ and let S be the portion of the cylinder $r = 2$ cut from the first octant and lying below the plane $z = 3$, with unit normal pointing away from the origin. Use Stokes' Theorem to write the flux of the curl of $\mathbf{F}$ across S as both a surface integral and a line integral. Draw S and its boundary C, oriented appropriately, and set up the two integrals using explicit parametrizations of S and C. *Extra credit:* Evaluate both integrals.

2. Let $\mathbf{F} = \langle 2xy - z^2, yz + 3x^2, 2y^2 - xz \rangle$ and let S be the cylindrical can $r = 2$ with top on the plane $z = 3$ and bottom on the xy-plane. Use the Divergence Theorem to write the outward flux of $\mathbf{F}$ across S as both a surface integral and a volume integral. Draw S and set up both integrals explicitly. *Extra credit:* Evaluate both integrals.