Math 3860, Spring 06 — Answers to the final exam

All questions are multiple-choice. You do not have to justify your answers.

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1. If $y^3 - y + 1 = Ce^t$, where C is constant, then $dy/dt =$

A. $3y^2 - 1$.

B. Ce^t.

C. $(3y^2 - 1) \cdot Ce^t$.

$\rightarrow$ D. None of the above.

2. $\displaystyle\int \frac{dt}{t^2 - 4t + 3} =$

A. $\log\left|t^2 - 4t + 3\right| + C$.

$\rightarrow$ B. $\frac{1}{2}\log|(t-3)/(t-1)| + C$.

C. $\log\left|\frac{1}{3}t^3 - 2t^2 + 3t\right| + C$.

D. None of the above.

3. If $y' + 3t^2 y = \cos(t)$ then $y =$

A. $\sin(t) + Ce^{-t^3}$.

B. $e^{t^3}\sin(t) + Ce^{t^3}$.

C. $e^{-t^3}(\sin(t) + C)$.

$\rightarrow$ D. None of the above.

4. If $y' = 2y - 3y^2$ then and y is not constant then $y =$

$\rightarrow$ A. $2/(3 - Ce^{-2t})$, for some constant C.

B. $t^2 - t^3 + C$, for some constant C.

C. $e^{2t} - e^{3t} + C$, for some constant C.

D. None of the above.

5. For the differential equation $y' = \sin(y)$ the stable equilibria are

$\rightarrow$ A. $y = \ldots, -3\pi, -\pi, \pi, 3\pi, \ldots$.

B. $y = \ldots, -2\pi, 0, 2\pi, 4\pi, \ldots$.

C. $y = \ldots, -\frac{1}{2}\pi, \frac{1}{2}\pi, \frac{3}{2}\pi, \ldots$.

D. None of the above.

6. The domain of validity for the solution of the initial value problem $y' = 2y - 3y^2$, $y(0) = 1$ is

A. $(0, 2/3)$.

→ B. $(-\ln\sqrt{3}, +\infty)$.

C. $(-\infty, +\infty)$.

D. None of the above.

7. The domain of validity for the solution of the initial value problem $(t^2 - 4)y' + (t - 1)y = \sqrt{t+1}$, $y(0) = -3$ is

A. $(-2, 1)$.

→ B. $(-1, 2)$.

C. $(-1, 1)$.

D. None of the above.

8. Any two solutions to a linear differential equation

A. differ by a constant of integration.

→ B. differ by a solution of the homogeneous equation.

C. differ by a solution of the nonhomogeneous equation.

D. None of the above.

9. The integral equation $y(t) = -3 + \int_2^t s\, y(s)\, ds$ is equivalent to the initial value problem

A. $y' = ty - 2$, $y(0) = -3$.

B. $y' = ty + 3$, $y(-3) = 2$.

→ C. $y' = ty$, $y(2) = -3$.

D. None of the above.

10. The local truncation error in the Euler-Heun method

A. decreases linearly in the step size.

B. decreases quadratically in the step size.

→ C. decreases cubically in the step size.

D. None of the above.

11. The global error in the Euler-Heun method

A. decreases linearly in the step size.

→ B. decreases quadratically in the step size.

C. decreases cubically in the step size.

D. None of the above.

12. The power series representation of $f(t) = (t-1)e^t$ at $t_0 = 0$ is

A. $1 + (t-1) + \frac{1}{2}(t-1)^2 + \frac{1}{6}(t-1)^3 + \frac{1}{24}(t-1)^4 + \cdots$.

→ B. $-1 + \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{8}t^4 + \cdots$.

C. $1 - t + \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{24}t^4 + \cdots$.

D. None of the above.

13. Any power series solution at $t_0 = 0$ for the differential equation $(t^2 - 4)y'' + ty' + 3y = 0$ has radius of convergence

A. less than or equal to 2.

B. exactly equal to 2.

→ C. greater than or equal to 2.

D. None of the above.

14. If we apply the Euler-Heun method to the initial value problem $y' = 2ty$, $y(0) = 1$ using a step size of 0.1 then the relative global error after one step is approximately

A. 1%.

B. 0.1%.

→ C. 0.005%.

D. None of the above.

15. If the phase portrait of the constant-coefficent system $\vec{w}' = P\vec{w}$ exhibits a saddle point then

A. $\text{tr}(P) < 0$.

→ B. $\det(P) < 0$.

C. $\text{tr}(P)^2 < 4\det(P)$.

D. None of the above.

16. If the phase portrait of the constant-coefficent system $\vec{w}' = P\vec{w}$ exhibits a stable spiral then

$\rightarrow$ A. $\text{tr}(P) < 0$ and $\text{tr}(P)^2 < 4\det(P)$.

B. $\text{tr}(P) < 0$ and $\text{tr}(P)^2 > 4\det(P)$.

C. $\text{tr}(P) < 0$ and $\det(P) < 0$.

D. None of the above.

17. If $y'' + 4y' + 5y = 0$, $y(0) = 2$, $y'(0) = -3$, the $y =$

A. $e^{-2t}(2\cos(t) + i\sin(t))$.

B. $e^{-2t}(2\cos(t) + \sin(it))$.

$\rightarrow$ C. $e^{-2t}(2\cos(t) + \sin(t))$.

D. None of the above.

18. If $y'' - 3y' + 2y = 2t^2 - 1$, $y(0) = 1$, $y'(0) = 1$, then $y =$

$\rightarrow$ A. $t^2 + 3t + 3 - 2e^t$.

B. $t^2 + 3e^{2t} - 2e^t$.

C. $t^2 + 3e^{2t} + 2e^{-t}$.

D. None of the above.

19. If $f(t) = t$, when $0 \le t < 1$, and $f(t) = 0$ otherwise, then $\mathcal{L}\{f(t)\} =$

A. $s^{-2} \cdot (1 - e^{-s})$.

B. $2s^{-2} \cdot (1 - e^{-s})$.

C. $s^{-1} \cdot (1 - e^{-s})$.

$\rightarrow$ D. None of the above.

20. If $Y(s) = s^{-1}/(1 + e^{-2s})$ then $\mathcal{L}^{-1}\{Y(s)\} =$

$\rightarrow$ A. $u_0(t) - u_2(t) + u_4(t) - u_6(t) + \cdots$.

B. $u_0(t) - 2u_1(t) + 4u_2(t) - 6u_4(t) + \cdots$.

C. $u_0(t) + u_2(t)e^{-2t} + u_4(t)e^{-4t} + u_6(t)e^{-6t} + \cdots$.

D. None of the above.